MATHEMATICAL MODEL OF HEAT AND MASS TRANSFER IN A MOVING BED OF DISPERSED MATERIAL

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A moving bed of moist dispersed material is treated as a continuous medium in investigation of the temperature field and as a discrete medium in the determination of the moisture content field. A simplified combined transport system is presented.

In the solution of a number of problems associated with the simulation of technological processes of heat and mass transfer it is necessary to consider a moving bed of dispersed material. Two approaches are possible: first, the universal, natural models of combined heat and mass transfer investigated by A. V. Luikov [1], based on which the problem has been formulated and solved for a fixed bed with allowance for the intraparticle potential distribution; and second, the approach in which the particles are assumed small enough so that it is possible to neglect the transport potential gradients, and to treat the bed as a continuum, the calculations being based on the equations of heat transfer with sources. In this case the sources are rather difficult to express analytically, especially when distributed, as a result of which it is more often than not necessary to resort to empirical formulas [2].

For some materials the temperature gradients are indeed so small that they can be neglected, whereas the moisture content gradients are not, with the moisture diffusion coefficient a_m, which depends on temperature and moisture content, playing a decisive part in the drying processes. Thus it is possible to write a simplified combined transport system, retaining for the temperature the equation of heat transfer with a distributed source without allowance for the heat fluxes associated with the thermal conductivity of the bed, and for the moisture content the equation of convective diffusion.

Starting from the internal energy transport equation [1]

$$\frac{\partial}{\partial \tau}\left(\sum_i \rho_i' h_i\right) = \operatorname{div}(\lambda' \nabla t) - \operatorname{div}\left(\sum_i h_i \mathbf{J}_i\right) + I_q,$$

where the summation is made over all the components, with allowance for the continuity equation

$$\frac{\partial \rho_i'}{\partial \tau} = -\operatorname{div} \mathbf{J}_i + I_i,$$

and the following equations:

$$\mathbf{J}_i = \rho_i' \mathbf{w}, \quad \rho_i' = u_i \gamma', \quad I_1 = -I_2,$$

we obtain

$$c'\gamma' \frac{\partial t}{\partial \tau} = \operatorname{div}(\lambda' \nabla t) - c'\gamma' \mathbf{w} \nabla t + I_q + \rho\left[\frac{\partial(\bar{u}\gamma')}{\partial \tau} + \mathbf{w}\nabla(\bar{u}\gamma')\right],$$

where $c' = \sum_i c_i u_i$ is the reduced specific heat of the dispersed material; $\bar{u} \approx \rho_2'/\gamma'$; $\gamma' = \gamma(1 - P)$; and $\mathbf{w} = \mathbf{w}(\tau)$. By definition [1]

$$I_q = \frac{q\Phi}{V} = \alpha F(t_m - t) \text{ for the material, and}$$

$$I_q^m = -\frac{q\Phi}{V} = \alpha F(t - t_m) \text{ for the medium.}$$

Neglecting the heat flows in the bed due to conduction as compared with the convective flows [2], we finally obtain

$$\frac{\partial t}{\partial \tau} + \mathbf{w}(\tau)\nabla t + \frac{\alpha F}{c'\gamma(1-P)}(t - t_m) - \frac{\rho}{c'}\left(\frac{\partial \bar{u}}{\partial \tau} + \mathbf{w}(\tau)\nabla \bar{u}\right) = 0. \quad (1)$$

On the basis of the continuity equation for the components of the individual particles

$$\frac{\partial \rho_i}{\partial \tau} = -\operatorname{div}(\mathbf{j}_i + \mathbf{J}_i) + I_i$$

with allowance for the equations

$$\rho_i = u_i \gamma, \quad \sum_i \frac{\partial u_i}{\partial \tau} \approx \frac{\partial u}{\partial \tau}$$

and the expression for the diffusion moisture flux [1]

$$\mathbf{j}_i = -a_m \gamma(\nabla u + \delta \nabla t),$$

neglecting shrinkage (i.e., for γ = const), we obtain the convective diffusion equation

$$\frac{\partial u}{\partial \tau} + \mathbf{w}\nabla u = \operatorname{div}(a_m \nabla u + a_m \delta \nabla t). \quad (2)$$

As a result of forced vapor convection through the particle bed the moisture concentration γ_1 in the vapor-gas medium is distributed in space. Using the continuity equation for the vapor mass, we obtain

$$\frac{\partial \gamma_1}{\partial \tau} + \mathbf{v}(\tau)\nabla\gamma_1 - \frac{q_m F}{P} = 0. \quad (3)$$

We now write the system of combined transport equations for a moving bed of dispersed material

$$\frac{\partial t}{\partial \tau} + \mathbf{w}(\tau)\nabla t + \frac{\alpha F}{c'\gamma(1-P)}(t - t_m) - \frac{\rho}{c'}\left(\frac{\partial \bar{u}}{\partial \tau} + \mathbf{w}(\tau)\nabla \bar{u}\right) = 0,$$

$$\frac{\partial t_m}{\partial \tau} + v(\tau)\nabla t_m + \frac{\alpha F}{c'_d \gamma_m P}(t_m - t) = 0,$$

$$\frac{\partial u}{\partial \tau} + w(\tau)\nabla u - \operatorname{div}(a_m \nabla u) = 0,$$

$$\frac{\partial \gamma_1}{\partial \tau} + v(\tau)\nabla \gamma_1 - \frac{q_m F}{P} = 0. \quad (4)$$

The convective diffusion equation is analogous to the Fourier-Kirchhoff equation used in studying heat and mass transfer in a binary vapor-gas mixture and in various processes of convective diffusion in fluids [1, 3]. In the case of a moving bed of capillary-porous particles, the convective moisture transport determines the spatial distribution of moisture content along the length of the drier.

The boundary conditions for Eq. (2) depend both on the choice of coordinate system and on the particle shape. If it is possible, the coordinate system should be selected to suit the problem. A rectangular Cartesian system is best for flat particles and a cylindrical system for cylindrical or spherical particles, since in a spherical coordinate system at large values of the coordinate r and small values of θ (as compared with the particle size) the Laplace operator takes the form corresponding to a cylindrical system. Particles of arbitrary shape should be simulated with cylindrical or flat particles, since in the case of axial motion the boundary conditions are most simply specified for such particles.

We consider the distribution of the temperature, field in an x, y Cartesian system. Then, for example, for a bed of cylindrical particles moving in the direction of the x-axis of symmetry and transverse motion (at 90°) of the drying agent, system (4) takes the form (disregarding axial motion transfer)

$$\frac{\partial t}{\partial \tau} + w(\tau)\frac{\partial t}{\partial x} + \frac{\alpha F}{c'\gamma(1-P)}(t - t_m) - $$
$$- \frac{\rho}{c'}\left(\frac{\partial \bar{u}}{\partial \tau} + w(\tau)\frac{\partial \bar{u}}{\partial x}\right) = 0,$$

$$\frac{\partial t_m}{\partial \tau} + v(\tau)\frac{\partial t_m}{\partial y} + \frac{\alpha F}{c'_d \gamma_m P}(t_m - t) = 0,$$

$$\frac{\partial u}{\partial \tau} + w(\tau)\frac{\partial u}{\partial x} - a_m\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = 0,$$

$$\frac{\partial \gamma_1}{\partial \tau} + v(\tau)\frac{\partial \gamma_1}{\partial y} - \frac{\beta\gamma F}{P}(u_R - u_p) = 0, \quad (5)$$

where $a_m = a_m(t)$, $\bar{u}(\tau, x, y) = \frac{2}{R^2}\int_0^R r u(\tau, x, y, r)\,dr$, $u_R = u(\tau, x, y, R)$, $u_p = u_p(t, \varphi)$, $\varphi = \frac{\gamma_1}{\gamma_1^{\max}(t_m, p_m)}$, with boundary conditions

$$t(\tau, 0, y) = f_1(\tau, y),$$
$$t_m(\tau, x, 0) = \varphi_1(\tau, x),$$
$$u(\tau, 0, y, r) = u^0(\tau, y, r),$$
$$\gamma_1(\tau, x, 0) = \chi_1(\tau, x),$$
$$\frac{\partial u}{\partial r}(\tau, x, y, R) = -\frac{\beta}{a_m}(u_R - u_p) \quad (6)$$

and initial conditions

$$t(0, x, y) = f_2(x, y),$$
$$t_m(0, x, y) = \varphi_2(x, y),$$
$$u(0, x, y, r) = u_0(x, y, r),$$
$$\gamma_1(0, x, y) = \chi_2(x, y). \quad (7)$$

The function u is assumed to be bounded on the axis of the cylindrical particle.

As pointed out in [4], the heat (α) and mass (β) transfer coefficients must be assumed to depend not only on the flow hydrodynamics, the physical properties of the gas, and the characteristic dimension of the flow surface but also on the time, the thermophysical properties of the body, the distribution of the heat sources, etc. This holds true even in the case of a "stationary" drier process (when the partial derivatives of the unknown variables with respect to time are zero), since owing to the dynamic nature of the system certain space variables play the part of time and the process is essentially nonstationary. Thus, the formulation of boundary value problem (5)–(7) is semiempirical and hence less natural than the formulation of the adjoint boundary value problem [4].

When a specific drier is considered, certain parameters (bed thickness, temperature, and relative humidity of drying agent at bed outlet, etc.) are known. Therefore, using model (5) to construct the control equation, it is first possible to average the unknown controlled variables with respect to certain space coordinates. This is done particularly easily for the temperature of the material and of the drying agent (at the bed outlet the latter is approximately equal to the material temperature) with respect to the y-coordinate.

Generally speaking, the functions entering into initial conditions (7) are arbitrary, but, for example, there may be some initial steady state (in the above-mentioned sense), from which the physical system (bed of dispersed material) departs in the presence of time-varying model parameters. In this case system (5) describes the transient process for spatially distributed temperature and moisture content. Although the given system of equations is simplified, it neglects temperature gradients, thermal diffusion, and the distributed nature of the heat source in the particle itself), it is quite complicated for analytical purposes (in view of the nonlinearity) and can be solved only by approximate computational methods. In the actual construction of a specific model (with corresponding constraints on the controlled and controlling variables, and with allowance for perturbations amenable to measurement) it is not necessary to know all the thermophysical coefficients entering into the model since, using the methods indicated, it is possible to adjust the model, utilizing only the form of the equations.

Based on Eqs. (4) or (5) adapted to existing moving-bed driers it is possible to construct a mathematical model of the drying process in such equipment and find the optimum operating and process control conditions, by using approximate numerical methods and a computer for solving the system of differential equations.

NOTATION

τ is the time; ρ_i' and ρ_i are the concentrations of the i-th component in the bed and the particle, respectively; h_i is the specific enthalpy of the i-th component; I_1 and I_2 are the vapor source and moisture sink, respectively; t is the temperature of the material; t_m is the temperature of the medium; **w** is the particle velocity; v is the drying agent velocity; α is the heat transfer coefficient; β is the mass transfer coefficient; Φ is the free particle surface in the volume V; F is the specific (per unit volume occupied by dispersed material) free particle surface; P is the porosity of the bed; q is the heat flux; q_m is the mass flux; c_i is the specific heat of the i-th component; γ' is the concentration of dry matter in moist dispersed material; γ is the concentration of dry matter in moist particle; γ_1 is the concentration of vapor in vapor-gas medium; γ_m is the concentration of gas in vapor-gas medium; c_d' is the reduced specific heat of the drying agent; p_m is the pressure of the vapor-gas medium; γ_1^{max} is the maximum water vapor concentration, possible at given temperature t_m and pressure p_m; ρ is the specific heat of vaporization; u is the moisture content of the material; a is the moisture diffusion coefficient; δ is the thermal gradient coefficient; and R is the radius of cylindrical particle.

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